Chatter Detection in Machine Milling by Signal Processing:
Empirical Mode Decomposition, hilbert Transform, & FFT Spectrum

Chatter is generally defined as self-generated vibrations from the interaction between tool and workpiece. It is the most critical vibration in machining operations and can decrease the surface quality and cause the premature tool wear. Chatter creates erroneous vibrations on the workpiece surface.

Methodology: ***Chatter Identification of Face Milling Operation via Time-Frequency and Fourier Analysis,***
Ching-Chih Wei, Meng-Kun Liu, and Guo-Hua Huang AuSMT Vol. 6 No. 1 (2016)
This mathcad worksheet is a signal processing technique of chatter detection for bar boring based on the hilbert–Huang Transform (HHT). HHT is suitable for the analysis of non-stationary and non-linear signals. The signals are **decomposed** into several **intrinsic mode functions (IMFs)** using empirical mode decomposition (**EMD**). **Dominant IMFs**, which best represent the occurance of chatter can be identified and combined to give the best **chatter indicator**. The **hilbert transform** is then applied to the chatter indicator **IMF** to obtain the instantaneous frequencies with time and their amplitudes. Finally, the marginal and the **hilbert spectrums** of strain signals were produced using selected IMFs.

The Reference Worksheets listed below are used in the Signal Processing of the Chatter data.
Note: The Array Origin in the referenced worksheets below is set to 1

Reference:C:\Users\Tom\Documents__Mathcad\Signal Analysis\EMD Extrema.xmcd(R)

Reference:C:\Users\Tom\Documents__Mathcad\Signal Analysis\EMD HHT Function.xmcd(R)

Read Two Sets of Data (A and B having Different depths of Cut) in 4 Columns (Time-microsec, Vibration-mv)

Milling_Strain := READPRN("Machine Milling Vibration Data.txt") $time := Milling\_Strain^{\langle 1\rangle}$ $S := Milling\_Strain^{\langle 2\rangle}$

$max(S) = 18$ $min(S) = -21$ $RS := rows(S) = 1303$

$rows(extrema(S)) = 1003$ $HHT := eemd(S,0,1)$ $cols(HHT) = 11$

$r := 1..RS$ $X_r := r$ $Residual := HHT^{\langle 6\rangle} + HHT^{\langle 7\rangle} + HHT^{\langle 8\rangle} + HHT^{\langle 9\rangle} + HHT^{\langle 10\rangle} + HHT^{\langle 11\rangle}$

$Ext := extrema(S)$ $RExt := rows(Ext)$ $spmax := submatrix(Ext,1,Ext_{RExt,2},1,2)$ $Ext_{RExt-1,2} = -4.5$

$spmin := submatrix(Ext,Ext_{RExt,2}+1,RExt-2,1,2)$ $up_{sp} := cspline(spmax^{\langle 1\rangle},spmax^{\langle 2\rangle})$

$upper := interp(up_{sp},spmax^{\langle 1\rangle},spmax^{\langle 2\rangle},X)$ Sample times in μs: $\frac{time_{RS}}{RS-1} = 768$ $T_s := 768\times 10^{-6}$

$low_{sp} := cspline(spmin^{\langle 1\rangle},spmin^{\langle 2\rangle})$ $lower := interp(low_{sp},spmin^{\langle 1\rangle},spmin^{\langle 2\rangle},X)$ $T_s 1000 = 0.768$

$timeB := Milling\_Strain^{\langle 3\rangle}$ $SB := Milling\_Strain^{\langle 4\rangle}$ $RSB := rows(SB) = 1303$ $max(SB) = 27$ $min(SB) = -22$

$rows(extrema(SB)) = 946$ $HHTB := eemd(SB,0,1)$ $cols(HHTB) = 11$ $i := 1..rows(HHTB)$

$r2 := 1..RSB$ $XB_r := r2$ $ResidualB := HHTB^{\langle 6\rangle} + HHTB^{\langle 7\rangle} + HHTB^{\langle 8\rangle} + HHTB^{\langle 9\rangle} + HHTB^{\langle 10\rangle} + HHTB^{\langle 11\rangle}$

$Ext2 := extrema(SB)$ $RExt2 := rows(Ext2)$ $spmax2 := submatrix(Ext2,1,Ext2_{RExt2,2},1,2)$ $Ext2_{RExt2-1,2} = -1$

$spmin2 := submatrix(Ext2,Ext2_{RExt2,2}+1,RExt2-2,1,2)$ $up2_{sp} := cspline(spmax2^{\langle 1\rangle},spmax2^{\langle 2\rangle})$

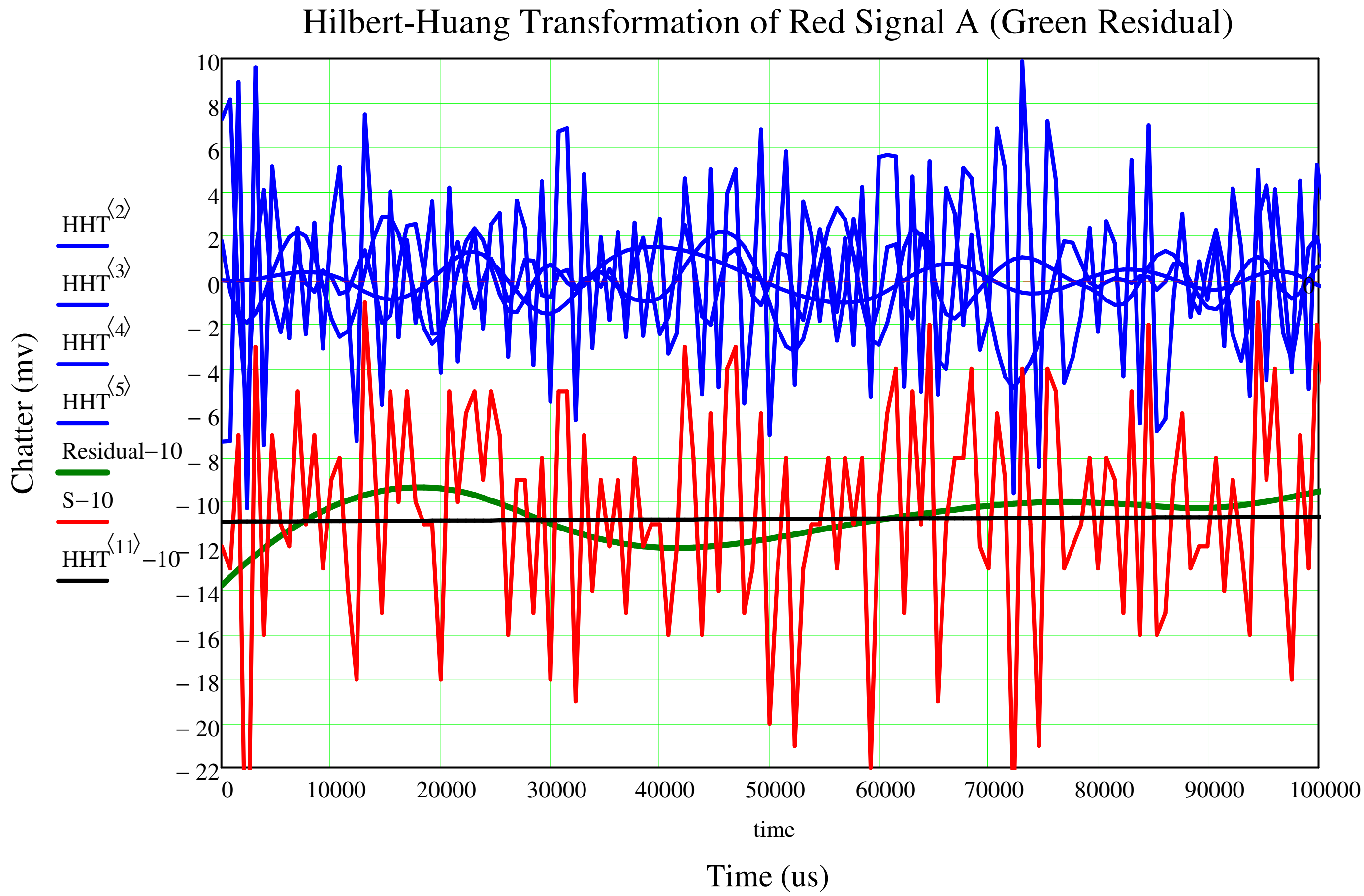
$upper2 := interp(up2_{sp},spmax2^{\langle 1\rangle},spmax2^{\langle 2\rangle},XB)$ $timeB_{RSB} = 999936$

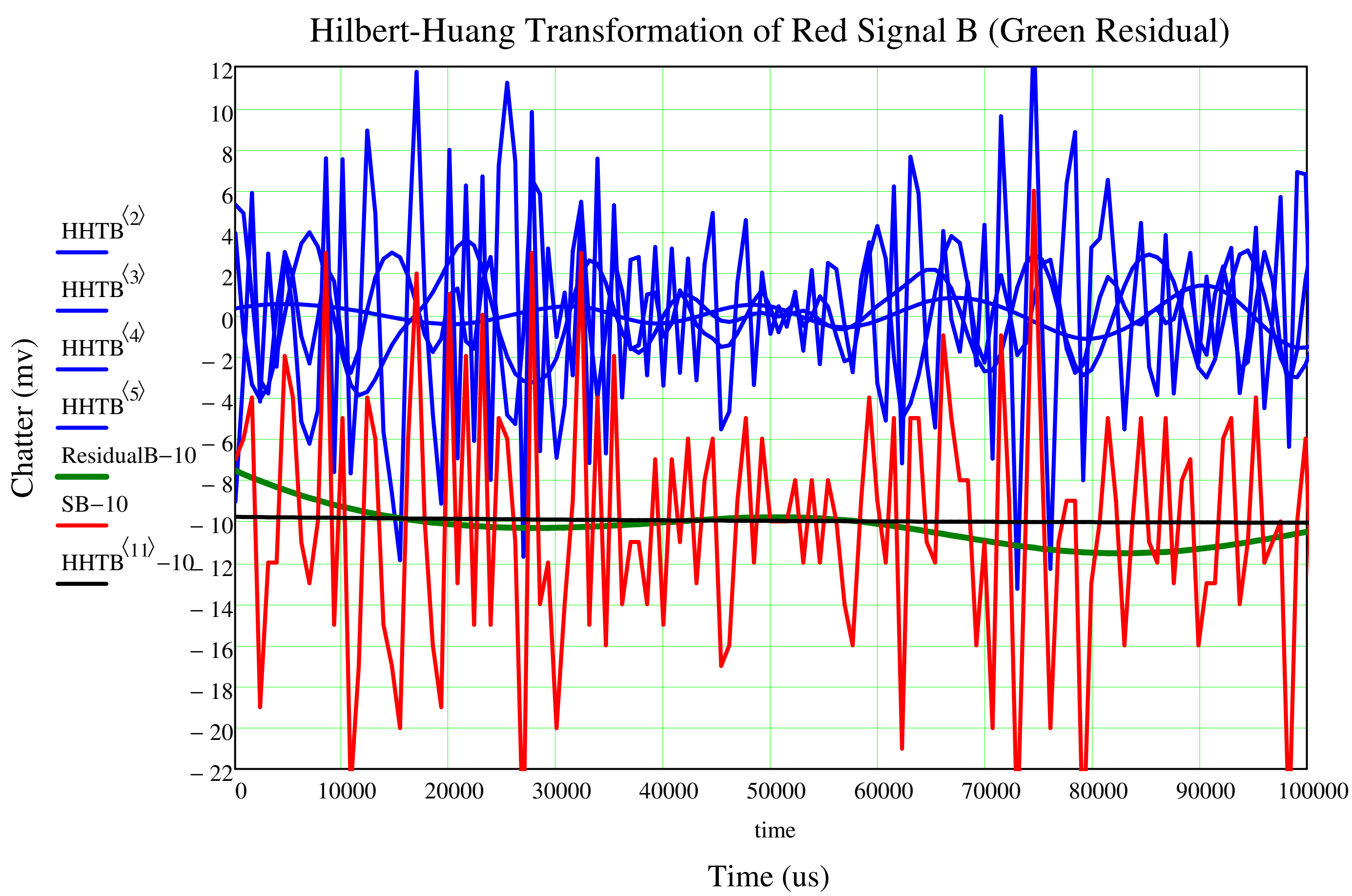
$low2_{sp} := cspline(spmin2^{\langle 1\rangle},spmin2^{\langle 2\rangle})$ $lower2 := interp(low2_{sp},spmin2^{\langle 1\rangle},spmin2^{\langle 2\rangle},XB)$

NOTE: The Array Origin of the worksheets is 1. HHT<1> is just the original signal.
Therefore HHT<5> is the Intrinsic Mode 4.

Empirical Mode Decomposition of Milling Chatter

Note: Signal and Residual is offset by -10 units to display input signal separately.
Displayed is a 0.1 second interval out of 1 second of data sampled at 768 μs intervals.





Ratio of Normalized Spectral Energy Content, NERatio

Index n starts with 2 so that the original EMD HHT function (n=1) output is excluded from the Ratio

$n := 2..11$

$$NERatio_n := \sum_{r=1}^{RS} \frac{\left(HHT_{r,n}\right)^2}{\sum_{r=1}^{RS} \left(S_r\right)^2} \qquad NERatioB_n := \sum_{r=1}^{RS} \frac{\left(HHTB_{r,n}\right)^2}{\sum_{r=1}^{RS} \left(SB_r\right)^2} \qquad \sum NERatio = 1.091 \qquad \sum NERatioB = 1.156$$

Comparison of Energy Ratios of Spectrum Content of B to A

$$ERatioBtoA_n := \frac{NERatioB_n}{NERatio_n}$$

Note: Below value of the Normalized Energy Ratio of Sample B to A of Intrinsic Mode 4 (HHT<5>, Z5) is 2.235.
See Graph of Z5 and ZB5 at bottom of next page.

Normalized Spectral Energy Content

$NERatio^T =$

	1	2	3	4	5	6	7	8	9	10
1	0	0.794	0.153	0.063	0.033	0.02	0.01	0.002	0.002	...

$NERatioB^T =$

	1	2	3	4	5	6	7	8	9	10
1	0	0.778	0.176	0.094	0.074	0.015	0.009	0.004	0.004	...

$ERatioBtoA^T =$

	1	2	3	4	5	6	7	8	9	10
1	0	0.98	1.153	1.485	2.235	0.757	0.915	1.856	2.116	...

Above ERatios show IMFs 3, 4, 5 have the greatest discrimination and are added for our chatter detector, IMFD.

$$IMFD := HHT^{\langle 3 \rangle} + HHT^{\langle 4 \rangle} + HHT^{\langle 5 \rangle} \qquad IMFDB := HHTB^{\langle 3 \rangle} + HHTB^{\langle 4 \rangle} + HHTB^{\langle 5 \rangle}$$

Signal Analysis of Intrinsic Mode Functions, IMFs, from Empirical Mode Decomposition

Compute the transform for **Modes 1 to 5**:

Build the complex signal:

Find the discrete Fourier transform spectrum CFFT of the complex signals in the frequency domain:

Data set A

$y2 := hilbert\left(HHT^{\langle 2 \rangle}\right)$	$z2 := HHT^{\langle 2 \rangle} + j\,y2$	$Z2 := CFFT(z2)$
$y3 := hilbert\left(HHT^{\langle 3 \rangle}\right)$	$z3 := HHT^{\langle 3 \rangle} + j\,y3$	$Z3 := CFFT(z3)$
$y4 := hilbert\left(HHT^{\langle 4 \rangle}\right)$	$z4 := HHT^{\langle 4 \rangle} + j\,y4$	$Z4 := CFFT(z4)$
$y5 := hilbert\left(HHT^{\langle 5 \rangle}\right)$	$z5 := HHT^{\langle 5 \rangle} + j\,y5$	$Z5 := CFFT(z5)$
$y6 := hilbert\left(HHT^{\langle 6 \rangle}\right)$	$z6 := HHT^{\langle 6 \rangle} + j\,y6$	$Z6 := CFFT(z6)$
$yd := hilbert(IMFD)$	$zd := IMFD + j\,yd$	$Zd := CFFT(zd)$

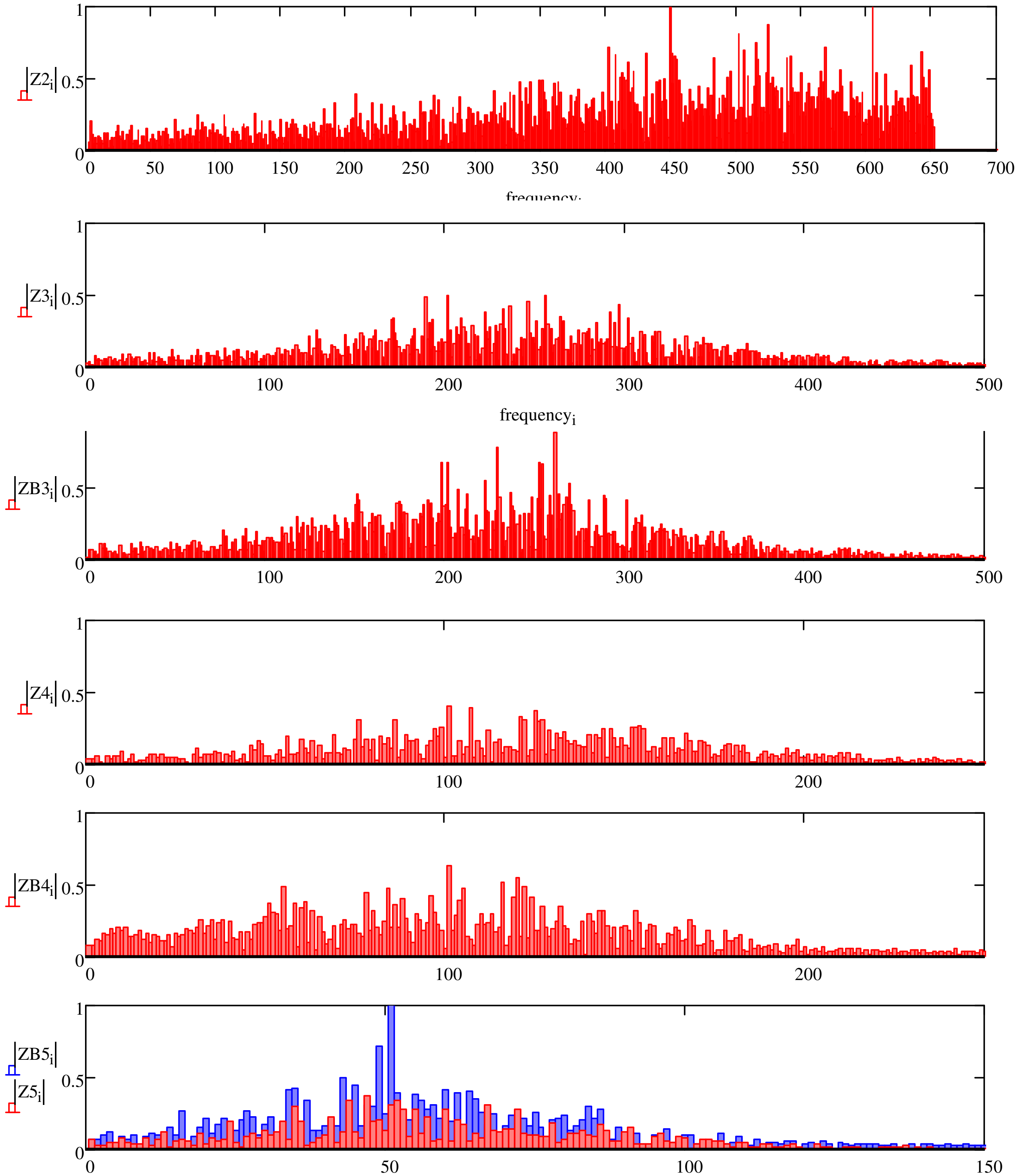
Data set B

$w2 := hilbert\left(HHTB^{\langle 2 \rangle}\right)$	$zB2 := HHTB^{\langle 2 \rangle} + j\,w2$	$ZB2 := CFFT(zB2)$
$w3 := hilbert\left(HHTB^{\langle 3 \rangle}\right)$	$zB3 := HHTB^{\langle 3 \rangle} + j\,w3$	$ZB3 := CFFT(zB3)$
$w4 := hilbert\left(HHTB^{\langle 4 \rangle}\right)$	$zB4 := HHTB^{\langle 4 \rangle} + j\,w4$	$ZB4 := CFFT(zB4)$
$w5 := hilbert\left(HHTB^{\langle 5 \rangle}\right)$	$zB5 := HHTB^{\langle 5 \rangle} + j\,w5$	$ZB5 := CFFT(zB5)$
$w6 := hilbert\left(HHTB^{\langle 6 \rangle}\right)$	$zB6 := HHTB^{\langle 6 \rangle} + j\,w6$	$ZB6 := CFFT(zB6)$
$wd := hilbert(IMFDB)$	$zBd := IMFDB + j\,wd$	$ZBd := CFFT(zBd)$

Sampling time, T_s in seconds: $T_s = 768\,10^{-6}$ $\qquad f_s := \frac{1}{T_s} \qquad frequency_i := \frac{i}{RS} f_s$

Comparison of Frequency Spectrum of Vibration Samples A versus B

Sample B (See Plots ZB2 and ZB3) displays more chatter than sample A.



Spectrum: Chatter Detector IMFD Signal. A is red and B is deeper cut blue signal.